Strings from Flux Tube Solutions in Kaluza-Klein Theory

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Abstract

We calculate dimensional reduction of gravitational flux tube solutions in the scheme of Kaluza-Klein theory. The fifth dimension is compacified to a region of Planck size. Assuming the width of the tube to be also Planck size we obtain string-like object with physical fields originated from an initial 5D metric. The dynamics of these fields is inner one.

1 Introduction

In string theory it assumed that strings are line-like objects without inner structure. Historically, strings arose in an attempt to explain the infinitelly rising Regge trajectories of hadrons. In field theory such objects were found in non-Abelian gauge theory in the form of Nielsen-Olesen flux tubes with a finite thickness. We would like to point out that in 5D Kaluza - Klein gravity there are vacuum solutions which have the form of strings [1].

Initially, the thickness of a string in quantum field theory is arbitrary since the classical theory is scale invariant. Quantum fluctuations, however, will fix a scale and string obtaines a thickness. The finite thickness gives such a string an extrinsic curvature stifness [2] [3] which is absent in the Nambu-Goto string Lagrangian.

Ordinary, string dynamics arises from a field equations for the movement in an external spacetime. Of coarse we do not have any another possibility because strings can have only such degrees of freedom as their coordinates. One can call such string dynamics as an external dynamics. The string-like objects derived here have an inner structure, and thus an inner dynamics : the dynamics of inner degrees of freedom. In this case we do not need for an external spacetime for the description of such kind of string dynamics. Ordinary, the fields on

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strings are introduced by hand but if strings has an inner structure then some fields will arise by the natural way as the consequence of the inner structure.

The basic goal of this paper is to show that for string-like objects considered here (gravitational flux tube solutions) the fields on string have the origin from the 5D metric and the dynamics follows from the 5D Einstein's equations.

2 Gravitational flux tube solutions

We begin with a brief reminder of the flux tube solutions [4] of the vacuum Kaluza-Klein gravity. The topology of these solutions is $M^2 \times S^2 \times S^1$ where M^2 is the 2D space-time spanned on the time and longitudinal coordinate r; S^2 is the cross section of this flux tube solution and it is spanned on the ordinary spherical coordinates θ and φ; $S^1 = U(1)$ is abelian gauge group which in this consideration is the 5^{th} dimension.

The possible reduction from a flux tube to strings is based on the following remark : the linear sizes of S^2 is not defined from Einstein's equations and can be arbitrary. Our strategy is that we set the radius of sphere S^2 to a Planck length l_{Pl}. In QCD the transversal sazes of flux tube stretched between quark-antiquark is an inner property : in principle it can be calculated inside of QCD. This gives us a possibility that after quantum fluctuations are taken into account, the calculated thickness of the gravitational flux tube will be of the order of the Planck scale.

Mathematically the cross section of the flux tube is not the point but physically it is indistiquishable with a point because one of the paradigms of quantum gravity claimes that the lengths less the Planck length have not any sense. Thus, from this physical point of view our flux tube solution after this reduction is like to string (of coarse we should have the length of the 5^{th} coordinate $\approx l_{Pl}$, too).

Now we can consider more carefully the flux tube solutions [4] in 5D Kaluza-Klein gravity. The 5D metric we take in the form

$$\begin{aligned} ds^2 &= e^{2\nu(r)}dt^2 - l_0^2 e^{2\psi(r)-2\nu(r)}\left[d\chi + \omega(r)dt + Q\cos\theta d\varphi\right]^2 \\ &- dr^2 - a(r)(d\theta^2 + \sin^2\theta d\varphi^2), \end{aligned} \tag{1}$$

where χ is the 5^{th} extra coordinate; r, θ, φ are $3D$ spherical-polar coordinates; Q is some constant; $r \in \{-R_0, +R_0\}$ (R_0 may be equal to ∞); l_0 is the radius of 5^{th} coordinate. $\omega(r)$ is the t-component of the electromagnetic potential and $Q\cos\theta$ is the φ-component. This means that we have radial Kaluza-Klein electrical E_r and magnetic $H_r \propto Q/r^2$ fields. The $R_{5\chi}$ equation

$$\omega'' - 4\nu'\omega' + 3\omega'\psi' + \frac{a'\omega'}{a} = \frac{e^{-3\psi+4\nu}}{4\pi a}\left(\omega' e^{3\psi-4\nu}4\pi a\right)' = 0. \tag{2}$$

give us

$$E_r = \omega' = \frac{q}{a}e^{4\nu-3\psi} \tag{3}$$

where q is some constant. From the $R_{5\chi}$ equation we see that it is like to the Maxwell equation in a continuous medium. In this case $e^{3\psi-4\nu}$ is like to a dielectric permeability and $\mathcal{D}_r = \omega' e^{3\psi-4\nu}$ is a dielectric displacement. The flux of electric field in this case is

$$\Phi = \mathcal{D}_r \times S = 4\pi a \omega' e^{3\psi-4\nu} = 4\pi q. \tag{4}$$

Consequently we can identify the constant q with an electric charge. One can note that it is not the point charge located in some point but this constant is an amount of the electric force lines. In Ref. [4] was shown that the solutions of the 5D Kaluza-Klein equations with this metric have the following qualitative behaviour

1. $0 \leq H_{KK} < E_{KK}$ (or $q > Q$). The solution is *the regular finite flux tube*. The throat between the surfaces at $\pm r_0$ is filled with both "electric" and "magnetic" fields. The longitudinal distance between the $\pm r_0$ surfaces depends on the relation between electric and magnetic fields.

2. $H_{KK} = E_{KK}$ (or $q = Q$). In this case the solution is *an infinite flux tube* filled with constant electric and magnetic fields. The cross-sectional size of this solution is constant ($a =$ const.).

3. $0 \leq E_{KK} < H_{KK}$ (or $q < Q$). In this case we have *a singular finite flux tube* located between two (+) and (-) electrical and magnetic charges located at $\pm r_0$. Thus the longitudinal size of this object is again finite, but now the cross sectional size decreases as $r \to r_0$. At $r = \pm r_0$ this solution has real singularities which we interpret as the locations of the charges.

Here r_0 defines the places where $ds^2 = 0$ ($\chi, \theta, \varphi =$ const, $r = \pm r_0$). Let us introduce a parameter $\delta = 1 - Q/q$. For us is interesting the first kind of solutions : finite and infinite flux tube for which we have $\delta \geq 0$. We can consider (for $\delta > 0$, $\delta \neq 0$) the limited region ($|r| \leq r_0$)of this solution for which $a \approx l_{Pl}^2$. In Ref. [1] it is shown that this part ($|r| \leq r_0$) of the flux tube solution ($|r| \leq \infty$) can be considered as a thin flux tube filled with the elctric and magnetic fields. According to the above mentioned reasonings it is a string-like object, for the care we can call his as a thread.

It is necessary to note that one can insert this part of flux tube solution between two Reissner-Nordström black holes [5]. This situation is much more interesting because it is like to string attached to 2 $D-$branes : the flux tube solution is the string, each Reissner-Nordström solution is $D-$brane and joining on the event horizon takes place. In this approach the string (dimensionally reduced flux tube solution) has a lightlike world sheet boundaries what is very close to $H-$branes approach introduced in Ref.[6].

3 Reduction of Flux Tube to String

Now we would like to reduce our initial 5D Lagrangian to 2D Lagrangian. At this step we set the sizes of $5^{\rm th}$ and S^2 dimensions $\approx l_{Pl}$. The first step is the usual 5D $\rightarrow$ 4D Kaluza-Klein dimensional reduction. Following, for example, to review [7] we have

$$\frac{1}{16\pi \overset{(5)}{G}} \overset{(5)}{R} = \frac{1}{16\pi G} \overset{(4)}{R} - \frac{1}{4}\phi^2 F_{\mu\nu}F^{\mu\nu} + \frac{2}{3}\frac{\partial_\alpha \phi \partial^\alpha \phi}{\phi^2} \tag{5}$$

where $\overset{(5)}{G} = G\int dx^5$ is 5D gravitational constant; G is 4D gravitational constant; $\overset{(5),(4)}{R}$ are 5D and 4D Ricci scalars respectively. The 5D metric has the following form

$$d\overset{(5)}{s}{}^2 = g_{AB}dx^A dx^B = g_{\mu\nu}(x^\alpha)dx^\mu dx^\nu - \phi^2(x^\alpha)\left(dx^5 + A_\mu(x^\alpha)dx^\mu\right)^2 \tag{6}$$

where $A, B = 0,1,2,3,5$ are 5D indices; $\mu,\nu = 0,1,2,3$ are 4D indices. The determinant of 5D and 4D metrics are connected as $\overset{(5)}{g} = \overset{(4)}{g}\ \phi$. We assume that the length of $5^{\rm th}$ dimension is $\approx l_{Pl}$. One of the paradigm of quantum gravity says us that it is a minimal length in the Nature. Physically it means that not any physical fields can have any structure inside this region. For us it means that these fields $f(x^\mu) \approx f(x^\mu + \delta x^\mu)$ if $\delta x^\mu \leq l_{Pl}$. These arguments allow us to say that the physical fields are not depend on the $5^{\rm th}$ coordinate (of coarse, if its length is $\approx l_{Pl}$) and consequently $\partial f/\partial x^\mu \approx 0$ inside Planck scale.

The next step is reduction from 4D to 2D. Let us remember that we consider the region of spacetime where the topology is $M^2 \times S^2 \times S^1$ and the linear sizes of S^2 are $\approx l_{Pl}$. As above we can conclude that all physical fields can not depend on the coordinates on the sphere S^2. For an external observer it is a physical (not mathematical !) point. Our statement is that *this point can not be colored by different colors*. The 4D metric can be expressed as

$$\begin{aligned} d\overset{(4)}{s}{}^2 &= g_{\mu\nu}dx^\mu dx^\nu = g_{ab}(x^c)dx^a dx^b + \\ &\chi(x^c)\left(\omega^{\bar{i}} + B^{\bar{i}}_a(x^c)dx^a\right)\left(\omega_{\bar{i}} + B_{\bar{i}a}(x^c)dx^a\right) \end{aligned} \tag{7}$$

where $a,b = 0,1$; x^a are the time and longitudinal coordinates; $-\omega^{\bar{i}}\omega_{\bar{i}} = dl^2$ is the metric on the 2D sphere S^2; all physical quantities g_{ab}, χ and $B_{\bar{i}a}$ can depend only on the physical coordinates x^a. Accoridingly to Ref. [8] we have the following dimensional reduction to 2 dimensions

$$\begin{aligned} \overset{(4)}{R} &= \overset{(2)}{R} + R(S^2) - \frac{1}{4}\Phi^{\bar{i}}_{ab}\Phi^{ab}_{\bar{i}} - \\ &\frac{1}{2}h^{ij}h^{kl}\left(D_a h_{ik}D^a h_{jl} + D_a h_{ij}D^a h^{kl}\right) - \nabla^a\left(h^{ij}D_a h_{ij}\right) \end{aligned} \tag{8}$$

where $\overset{(2)}{R}$ is the Ricci scalar of 2D spacetime; D_μ and $\Phi^{\bar{i}}_{ab}$ are, respecively, the covariant derivative and the curvature of the principal connection $B^{\bar{i}}_a$ and $R(S^2)$ is the Ricci scalar of the sphere $S^2 = \mathrm{SU}(2)/\mathrm{U}(1)$ with linear sizes $\approx l_{Pl}$; h_{ij} is the metric on $\mathcal{L}$

$$\begin{aligned} \mathrm{su}(2) &= \mathrm{u}(1) \oplus \mathcal{L}, && (9) \\ \mathrm{su}(2) &= \mathrm{Lie}(\mathrm{SU}(2)), && (10) \\ \mathrm{u}(1) &= \mathrm{Lie}(\mathrm{U}(1)) && (11) \end{aligned}$$

here $\mathcal{L}$ is the orthohonal complement of u(1) algebra in su(2) algebra; the index $i \in \mathcal{L}$. The metric h_{ij} is proportional to the scalar χ in Eq. (7).

Now we would like to consider the situation with the electromagnetic fields A_μ and $F_{\mu\nu}$.

$$\begin{aligned} A_\mu &= \{A_a, A_i\} \ \ A_a \text{ is the vector; } A_i \text{ are 2 scalars;} && (12) \\ F_{ab} &= \partial_a A_b - \partial_b A_a \text{ is the Maxwell tensor for } A_a; && (13) \\ F_{ai} &= \partial_a A_i, && (14) \\ F_{ij} &= 0 && (15) \end{aligned}$$

here we took into account that $\partial_i = 0$ as *the point cannot be colored*.

Connecting all results we see that only the following *physical fields* on the our string-like object (thread) are possible : 2D metric g_{ab}, gauge fields $B^{\bar{i}}_a$, vectors A_a, tensors $F_{ab}, F_{a\bar{i}}, \Phi^{\bar{i}}_{ab}$, scalars χ and ϕ.

4 Discussion

The gravitatonal flux tube solutions with the cross section on the Planck level are interesting objects. In the longitudinal direction they have classical properties but in the transversal directions they become quantum mechanical. In fact, it is some mixture of classical and quantum characteristics in one object ! For example, on the thread we can investigate such extra complicated quantum object as spacetime foam only in two dimensions. Also in the case of small perturbations on the background flux tube metric the transport of energy through this string-like object from one part of Universe to another becomes possible and so on.

Another interesting property in this approach is that the string-like objects are the objects in the spirit of the Kaluza - Klein paradigm in gravity, *i.e.* a dimensional reduction of some initial high dimensional pure gravitational spacetime to another low-dimensional one.

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